Chaos Prediction in an MMIC Frequency Divider in Millimetric Band

Almudena Suárez, *Member, IEEE*, José Morales, *Student Member, IEEE*, and Raymond Quéré, *Member, IEEE*

***Abstract*—In this letter, a technique is proposed for the analysis of possible quasi-periodic routes to chaos in microwave circuits. This is based on the systematic application of a Nyquist stability analysis along the solution paths in self-oscillating mixer operation, as a function of a relevant parameter. The steady quasi-periodic solutions, with two fundamental frequencies, are determined from the harmonic balance method. The stability analysis allows the detection of possible asynchronous instabilities, leading to a three-fundamental quasi-periodic solution, from which the transition to chaotic behavior will occur. This technique has allowed the theoretical prediction of the onset of chaos that was experimentally observed in a monolithic microwave integrated circuit (MMIC) frequency divider by two in millimetric band.**

***Index Terms*—Chaos, frequency divider, local stability analysis, quasi-periodic regime.**

I. Introduction

THE analog frequency dividers, ruled by nonlinear dynamic equations, can exhibit different types of behavior according to the value of their parameters [1]. Thus, the frequency division will only be observed for certain ranges of the input generator amplitude and frequency. For other generator values, a multiplying operating mode without frequency division, a self-oscillating mixer mode, or even a chaotic regime may be obtained. The chaotic behavior is characterized by a continuous spectrum and a sensitive dependence on the initial conditions that makes the time evolution of the circuit solution unpredictable [2].

There are different routes, or bifurcation sequences, leading to a chaotic behavior when a parameter is modified [2]. Two of the best known are the period-doubling and the quasi-periodic route [3]. The latter consists of a transition from a quasi-periodic state with two incommensurate frequencies (a flow on a 2-torus) to a quasi-periodic state with three incommensurate frequencies (3-torus). It can be shown [2] that the perturbed dynamical system has a chaotic attractor in the neighborhood of the 3-torus, so chaos could arise from any small perturbation. Thus three fundamental states have seldom been observed [2].

The self-oscillating operating mode exhibited by harmonic injection dividers is in fact an autonomous quasi-periodic regime with a 2-torus flow, which is analyzed here by means of harmonic balance (HB). From this operating mode, the detection of possible asynchronous instabilities would allow the prediction of three-fundamental regimes and thus of chaotic behavior. In the present work, this analysis is carried out by perturbing the quasi-periodic steady states with a complex frequency, nonharmonically related to the two independent fundamentals. As the perturbation is small, the nonlinear circuit may be linearized about the quasi-periodic steady state. Then the Nyquist stability criterion [4] is applied.

The proposed technique is used here to investigate the behavior of a monolithic microwave integrated circuit (MMIC) frequency divider by two 28–14 GHz [3] that exhibited a chaotic response for some input generator values. The accuracy of the quasi-periodic analysis was initially checked by comparing simulated and measured spectra. When applying the Nyquist criterion to the quasi-periodic paths, the asynchronous instability prediction showed a very good agreement with the experimental generator values for the onset of chaos.

II. Analysis of the Quasi-Periodic Route to Chaos

In the low-input power range, harmonic injection dividers exhibit a self-oscillating mixer regime, outside the frequency division bands [1]. The solution will be quasi-periodic with two independent, nonharmonically related fundamentals, corresponding to a 2-torus flow. These fundamentals will be the external generator frequency ω_{in} and the autonomous frequency ω_a. This regime is analyzed by means of the HB technique. In a matrix form, the HB equations may be written as

$$[Ax]\bar{X} + [Ay]\bar{Y}(\bar{X}) + [Ag]\bar{G} = 0 \tag{1}$$

where $\mathbf{X}$ is the vector containing the spectral components of the control variables, $\mathbf{Y}$ the vector containing the spectral components of the nonlinearities, and $\mathbf{G}$ the vector containing the generator values at their corresponding frequencies. The matrices $[Ax]$, $[Ay]$, and $[Ag]$ are obtained from the analysis of the linear part of the circuit.

For self-oscillating mixer operation, the resolution of (1) is carried out by means of probe at the autonomous fundamental ω_a, as described in [1]. The quasi-periodic solution will be given by

$$x_s^k(t) = \sum_{m,n} X_{m,n}^k e^{j(m\omega_{\mathrm{in}} + n\omega_a)t} \tag{2}$$

with m, n integers and $1 \leq k \leq n_x, n_x$ being the number of control variables.

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A. Suárez and J. Morales are with the Department Ingeniería de Comunicaciones, Universidad de Cantabria, 39005 Santander, Spain (e-mail: almu@dicom.unican.es).

R. Quéré is with IRCOM Université de Limoges, Brive, France (e-mail: quere@brive.unilim.fr).

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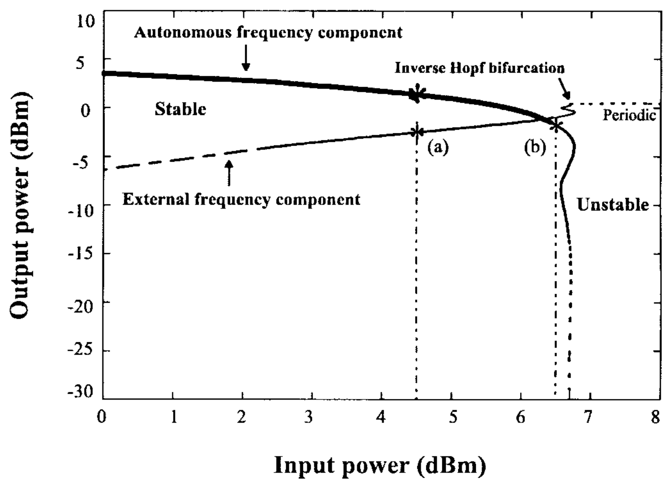

Fig. 1. Bifurcation diagram for constant input frequency 31.7 GHz.

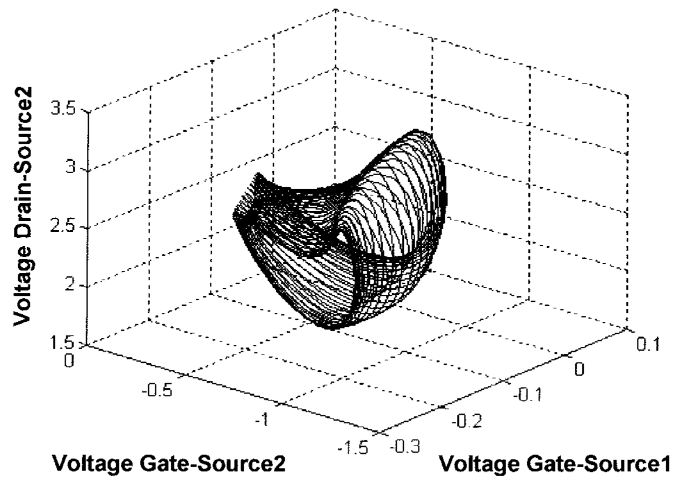

Fig. 2. Two torus flow of the MMIC frequency divider for input power $P_{\rm in} = 1$ dBm. Projection on the state space defined by the *gate-to-source* voltage of each of the two transistors [6] and the *drain-to-source* voltage of the one in the second stage.

From a two-fundamental quasi-periodic regime, as the input generator amplitude is modified, different bifurcations may take place. The relevant one for the quasi-periodic route to chaos will be the Hopf bifurcation in the direct sense. This corresponds to the appearance of a natural frequency, nonharmonically related to the two fundamentals $\omega_{\rm in}, \omega_a$. For the stability analysis of (2), a small perturbation will be considered [4]:

$$e^{(\sigma + j\omega t)}$$

with $\omega \neq m\omega_{\rm in} + n\omega_a$.

As the perturbation is small, the nonlinearity vector $\mathbf{Y}$ may be expanded in a first-order Taylor series around the steady solution. The HB system is then rewritten at the perturbed frequencies, applying the Nyquist criterion to the characteristic determinant

$$\begin{aligned} \det(\omega) = \det\{&[Ax(m\omega_{\rm in} + n\omega_a + \omega)] \\ &+ [Ay(m\omega_{\rm in} + n\omega_a + \omega)][JY]_s\} \end{aligned} \tag{3}$$

where $[JY]_s$ is the nonlinear functions jacobian matrix.

Due to the absence of poles of the characteristic determinant [4], an unstable natural frequency will correspond to a clockwise encirclement around the origin of the function (3), when ω is swept between $-\infty$ and ∞ [4]. According to [5], the real part ω of the perturbation frequency may be estimated from the frequency value ω_C at which Nyquist plot $\det(\omega)$ intersects the negative real axis.

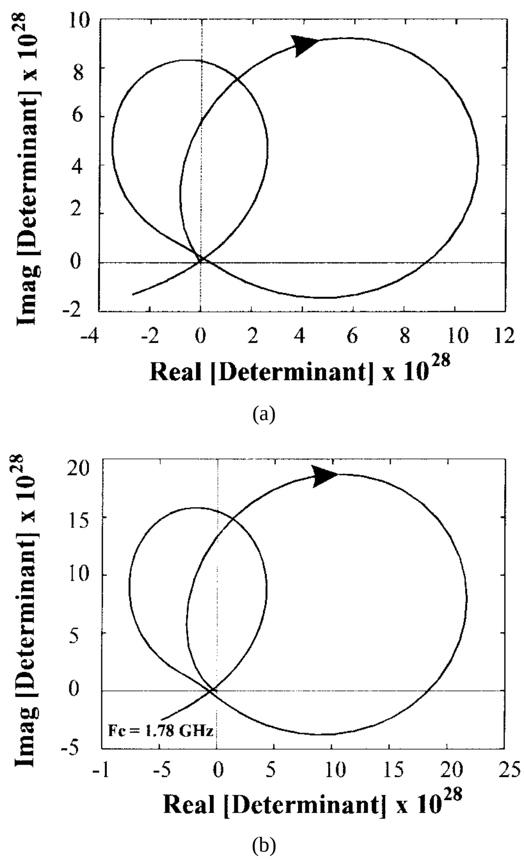

Fig. 3. Nyquist stability plots. (a) Input power 4.5 dBm. (b) Input power 6.5 dBm.

III. Results and Measurements

A quasi-periodic route to chaos was experimentally observed in the MMIC frequency divider by two of [6]. For 5-V self-bias and a constant input frequency $f_{\rm in} = 31.5$ GHz, this circuit operates in a self-oscillating mixer mode in the low-input power range. The evolution of the quasi-periodic response as a function of input power is shown in Fig. 1, where each path corresponds to one of the two independent fundamentals. The 2-torus flow, obtained for input power $P_{\rm in} = 1$ dBm and 5-V self-bias, is shown in Fig. 2. According to Fig. 1, the extinction of this regime should be due to an inverse Hopf bifurcation (disappearance of ω_a) for $P_{\rm in} = 6.7$ dBm. However, the systematic application of the Nyquist stability analysis [4] predicts an asynchronous instability for $P_{\rm in} = 6.5$ dBm (Fig. 3). It should be noticed that the existence of an autonomous frequency in the quasi-periodic regime implies that the determinant $\det(\omega)$ will be singular at frequency zero. The appearance of a second autonomous fundamental would make the spectrum denser, giving rise

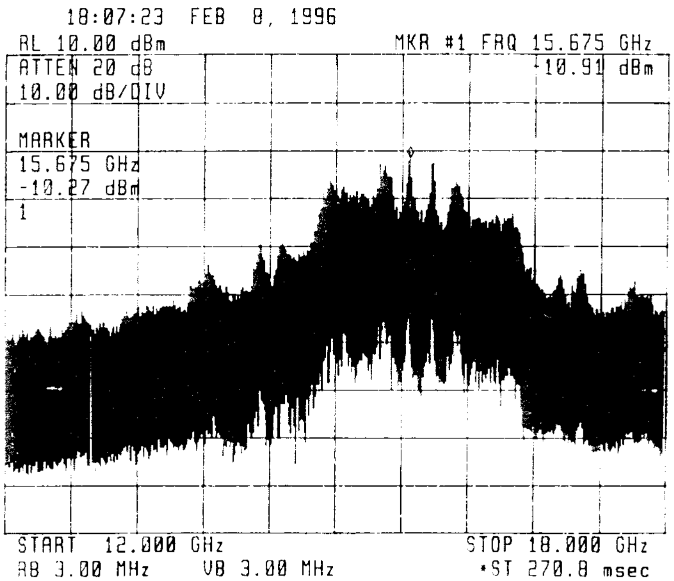

Fig. 4. Chaotic spectrum for input power 4.2 dBm.

to new spectral lines between the frequency components of the unstable two-fundamental regime. Thus, the sweeping interval may be reduced to $[-f_l, f_l]$, with f_l being the lowest intermodulation frequency component of the two-fundamental regime. Taking also into account the symmetry properties [4] of the Nyquist locus for negative perturbation frequency, the practical sweeping interval may be limited to $[0, f_l]$. The unstable locus of Fig. 3(b) intersects the negative real axis for $f_C = 1.74$ GHz. For $P_{\text{in}} = 6.5$ dBm, the autonomous fundamental of the 2-torus flow is $f_a = 14.98$ GHz, so the second autonomous fundamental may be expected to have a value $f'_a = 14.98 \pm 1.74$ GHz. This three-fundamental regime is not likely to be observed, as it would immediately break into chaos [2]. This was confirmed by the experiment. Fig. 4 shows the measured spectrum for $P_{\text{in}} = 4.2$ dBm, just after the onset of chaos. The difference between the simulation prediction and the experiment is thus only about 2 dB.

IV. Conclusion

In this letter the existence of quasi-periodic routes to chaos in harmonic injection dividers has been shown. The initial two-fundamental quasi-periodic regime, corresponding to a self-oscillating mixer operating mode, is simulated by means of harmonic balance. Then a systematic Nyquist stability analysis along the quasi-periodic paths is carried out. The stability analysis predicts the possible appearance of a second autonomous fundamental, leading to a very unstable three-fundamental regime, close to chaotic behavior. Through this technique, the chaotic behavior experimentally observed in a MMIC frequency divider by two, with 28-GHz input frequency, has been predicted by means of simulation.

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